

# Generation and detection of non-Abelian matrix Berry phases through manipulation of electric confinement potential of a semiconductor quantum dot

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A matrix Berry phase can be generated and detected by *all electric means* in II-VI or III-V n-type semiconductor quantum dots by changing the shape of the confinement potential. This follows from general symmetry considerations in the presence of spin-orbit coupling terms. The resulting  $2 \times 2$  matrix Berry phase can be characterized by two numbers of geometric origin. We investigate how these parameters depend on the shape and area of closed adiabatic paths. We suggest how the matrix Berry phase may be detected in transport measurements.

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## I. INTRODUCTION

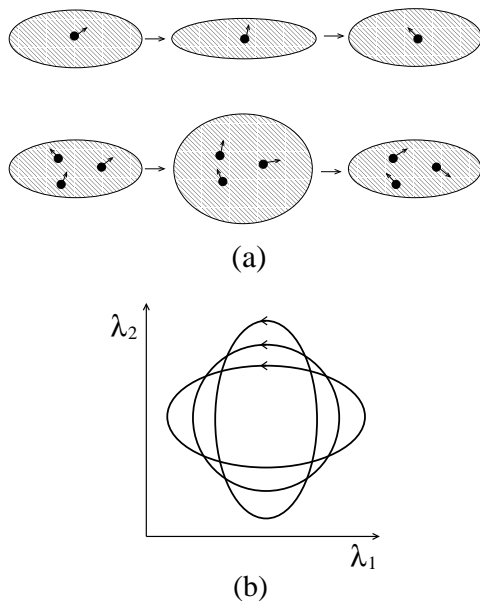


FIG. 1: (a) Displays electrons in quantum dots. A cyclic adiabatic change of the shape of the electric confinement potential does not return electron spins to the initial values. (b) Several adiabatic paths are shown. Adiabatic parameters  $\lambda_1$  and  $\lambda_2$  provides a means to control the shape of the dot electrically.

Single and few electron control by electric means in semiconductor quantum dots would be valuable for spintronics, quantum information, and spin qubits[1, 2, 3]. Adiabatic time evolution of degenerate eigenstates of a quantum system provides a means for controlling individual quantum states through the generation of non-Abelian matrix Berry phases[4, 5, 6]. This method

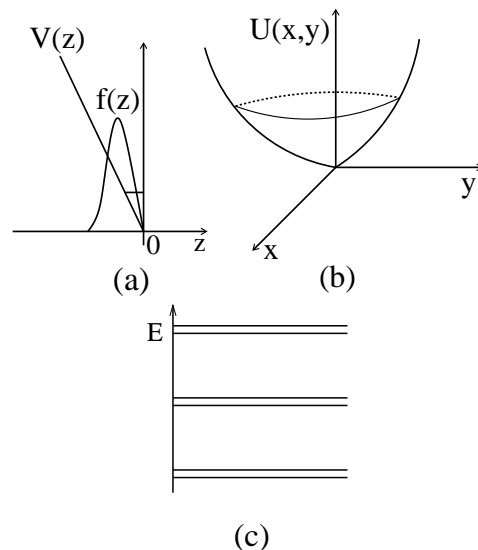


FIG. 2: Basic mechanism of adiabatic control is based on non-trivial degeneracy of II-VI and III-V semiconductors. (a) An electric field along the  $z$ -axis quantizes the electronic motion in a triangular potential along the axis. An adiabatic change can be induced by changing the magnitude of the electric field. (b) The two-dimensional electronic motion is quantized in a distorted parabolic potential. An adiabatic change can be induced by changing the magnitude of the distortion potential. (c) In the presence of the spin-orbit terms each discrete eigenstate of a semiconductor quantum dot has a double degeneracy due to time reversal symmetry in the absence of a magnetic field.

can be used to perform universal quantum computation [7]. There are several semiconductor nanosystems that exhibit matrix Berry phases: they include excitons[8], CdSe nanocrystals[9], and acceptor states of p-type semiconductors[10]. It is desirable to generate and detect matrix Berry phases by *all electric means* in semiconductor nanosystems. Recently we have demonstrated theoretically that it is possible to control *electrically* electron spins of II-V and III-V n-type semiconductor quantum dots[11, 12] and rings[13] by changing the

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shape of the electric confinement potential. The mechanism is based on spin orbit couplings which generate non-Abelian vector potentials. The main ingredients of it are: [a] Time-reversal symmetry, which leads to double degeneracy[11, 14], which is depicted schematically in Fig.1. [b] Non-Abelian U(2) gauge theory. [c] Adiabatic changes that break the parity symmetry of the electric confinement potential of the dot. When several electrons are present the matrix Berry phase can be generated only when odd number of electrons are in the dot. In such a case the effect of correlations can be included in a compact way (exchange effects do not affect the non-Abelian vector potentials)[12]. It should be emphasized that the presence of the matrix Berry phase follows from general symmetry considerations. The matrix Berry phase can be present even in the absence of the spin-orbit terms. A noteworthy example is semiconductor quantum dot pumps[16, 17], which can be understood as a manifestation of a matrix Berry phase[15].

Let us give a brief explanation of matrix Berry phases. The electron state at time  $t$  is given by

$$|\Psi(t)\rangle = c_1(t)|\psi_1(t)\rangle + c_2(t)|\psi_2(t)\rangle, \quad (1)$$

where  $|\psi_{1,2}(t)\rangle$  are the instantaneous basis states satisfying  $H(t)\psi_i(t) = E(t)\psi_i(t)$  for  $i = 1, 2$  ( $H(t)$  is the Hamiltonian with the eigenenergy  $E(t)$  at time  $t$ ). The matrix Berry phase  $\Phi_C(1)$  connects  $(c_1(T), c_2(T))$  to  $(c_1(0), c_2(0))$ , and is given by

$$\begin{aligned} \Phi_C(1) &= e^{i \sum_p A_p(t_n) d\lambda_p} \dots e^{i \sum_p A_p(t_1) d\lambda_p} \\ &= P e^{i \oint_C \sum_p A_p d\lambda_p}, \end{aligned} \quad (2)$$

where time slices are ordered as  $t_1 < \dots < t_n < \dots$ . The matrix vector potentials  $A_p$

$$(A_p)_{i,j} = i \langle \psi_i | \frac{\partial \psi_j}{\partial \lambda_p} \rangle, \quad (3)$$

are integrated along the path  $C$  in the parameter space in the order of increasing time. The path  $C$  is parameterized in time as  $(\lambda_1(t), \lambda_2(t))$ . The matrix Berry phase is independent of the functions  $\lambda_i(t)$  as long as they describe the same path  $C$ .

In applying matrix Berry phases to II-VI and III-V semiconductor quantum dots there are several issues that need to be addressed. One issue is whether the line integral along  $C$  can be *generally* converted into an areal integration over  $A$  in the expression for the matrix Berry phase. For this purpose it is useful to consider the field strength 2-form

$$F = \sum_{\mu\nu} F_{\mu\nu} d\lambda_\mu \wedge d\lambda_\nu, \quad (4)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \quad (5)$$

with  $\partial_\mu A_\nu = \frac{\partial A_\nu}{\partial \lambda_\mu}$ . For a small area  $A$  it can be shown[5] that the matrix Berry phase is

$$\Phi_C(1) = \exp(iAF_{12}). \quad (6)$$

For an arbitrarily large  $A$  there are two cases:[1] If  $[A_\mu, A_\nu] = 0$ , the curvature reduces to  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and we can use Stokes' theorem to convert the line integral along  $C$  to an areal integration over  $A$

$$\Phi_C(1) = \exp(i \frac{1}{2} \int_A F_{\mu\nu} d\lambda_\mu \wedge d\lambda_\nu). \quad (7)$$

[2] When  $[A_\mu, A_\nu] \neq 0$  it is not possible to convert the line integral into an areal integration. We will investigate the applicability of Stokes' theorem in the case of Rashba and Dresselhaus terms.

Second issue is that it is often complicated to calculate the non-Abelian vector potentials, and, consequently, it is difficult to compare experimental and theoretical results. This is especially so when many electrons are present. It would be useful to have a simple, yet general, mathematical expression for the matrix Berry phase. We will show that the matrix Berry phase is a unitary  $2 \times 2$  matrix and that it may be thought of as a spin rotation matrix[18]. A spin rotation matrix is characterized by the direction and angle of rotation. It is unclear how the geometric properties of the adiabatic path are reflected on them. We show in this paper how the information of the path  $C$  can be encoded in the direction and angle of the rotation.

Another issue is whether the matrix Berry phase can be detected in a transport measurement, which would complement the detection of matrix Berry phases by infrared optical measurements[11].

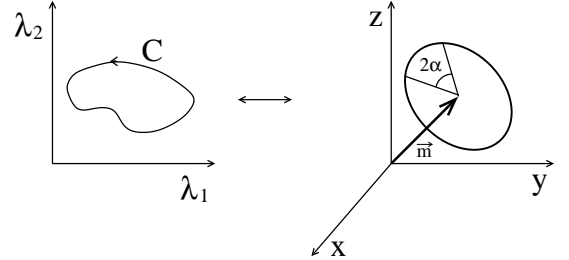


FIG. 3: The geometric information about the closed curve in the parameter space is encoded into the rotation axis  $\vec{m}$  and angle  $2\alpha$ . When only the Rashba term is present  $\vec{m}$  is independent of path  $C$ .

Let us now give a brief summary of the main results of this paper. First, the matrix Berry phase has the form

$$\Phi_C(n) = e^{\frac{i}{2}(2\alpha n)\vec{m} \cdot \vec{\sigma}} = \cos(\alpha n)I + i \sin(\alpha n)\vec{m} \cdot \vec{\sigma}, \quad (8)$$

where  $\vec{\sigma}$  are Pauli spin matrices and  $n$  is the number of periodic adiabatic cycles. The information about the

closed curve in the parameter space is encoded into the rotation axis

$$\vec{m} = (\text{Re}(\beta), -\text{Im}(\beta), \sqrt{1 - |\beta|^2}) \quad (9)$$

and the angle of rotation  $2\alpha n$ . The constants  $\alpha$  and  $\beta$  are real and complex numbers, respectively. This result is even valid for many electron systems with correlations as long as double degeneracy is present. The expression Eq.(8) is valid *only* at time  $t = nT$ , and it should be stressed that in the time interval  $(n-1)T < t < nT$  the probability amplitudes  $c_1(t)$  and  $c_2(t)$  exhibit a much more complicated behavior, see Fig.7. Second, we find  $\beta = 1$  and  $\vec{m} = (1, 0, 0)$  when only the Rashba term is present, and all the geometric information is contained in the rotation angle  $\alpha$ . We have derived an analytical expression for  $\alpha$ . Furthermore, remarkably  $[A_\mu, A_\nu] = 0$  and Stokes' theorem can be applied. When both Rashba and Dresselhaus terms are present each path has different  $\vec{m}$  and  $\alpha$ . We have investigated how they depend on the shape of each path. Moreover, we find that  $[A_\mu, A_\nu] \neq 0$  and that Stokes' theorem cannot be applied. Third, we propose how the presence of the matrix Berry phase can be measured in transport experiments by changing the electric confinement potential suddenly, see Sec.III. This is shown schematically in Fig.4.

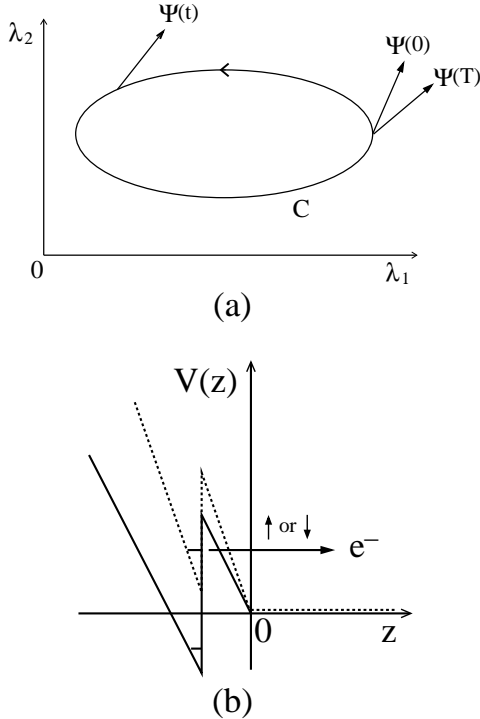


FIG. 4: (a) A matrix Berry phase is generated when the adiabatic parameters  $\lambda_1$  and  $\lambda_2$  go through a cyclic change. (b) After a matrix Berry phase is generated the Rashba electric field is suddenly increased so that an electron can tunnel out the dot.

## II. SEMICONDUCTOR QUANTUM DOTS WITH THE RASHBA AND DRESSSELHAUS SPIN ORBIT COUPLINGS

### A. Model

The Hamiltonian is  $H = -\frac{\hbar^2 \nabla^2}{2m^*} + U(x, y) + V(z) + H_R + H_D$ . We take the two-dimensional potential to be  $U(x, y) = \frac{1}{2}m^*\omega_x^2 x^2 + \frac{1}{2}m^*\omega_y^2 y^2 + \epsilon'y$ , where the term  $\epsilon'y$  represents a distortion of the two-dimensional harmonic potential. The strengths of the harmonic potentials are denoted by  $\omega_x$  and  $\omega_y$ . The characteristic lengths scales along x- and y-axis are  $R_{x,y} = \sqrt{\frac{\hbar}{m^*\omega_{x,y}}}$ . In our work the triangular potential  $V(z)$  is sufficiently strong and only the lowest energy subband is included. The characteristic length scale along the z-axis is  $1/\sqrt{0.8(2meE/\hbar^2)^{2/3}}$ , where  $E$  is the Rashba electric field applied along the z-axis. The Rashba spin orbit term[19, 20] is

$$H_R = c_R (\sigma_x k_y - \sigma_y k_x), \quad (10)$$

and the Dresselhaus spin orbit term[21] is

$$H_D = c_D ((\sigma_x k_x (k_y^2 - k_z^2)) + (\sigma_y k_y (k_z^2 - k_x^2))). \quad (11)$$

Note that the Rashba spin orbit constant  $c_R$  depends on the external electric field  $E$ . Here  $k_{x,y}$  are momentum operators ( $k_x = \frac{1}{i} \frac{d}{dx}$  and similarly with  $k_y$ ). The Hamiltonian matrix is represented in the basis states  $|mn\sigma\rangle$  of the harmonic oscillator states in the xy plane with spin component  $\sigma$ . The subband wavefunction  $f(z)$  is suppressed in the notation  $|mn\sigma\rangle$ . Let us take  $\omega_x = 2\omega_y$  (Other values of  $\omega_x$  can also be chosen). Then the lowest eigenenergy state of  $H$  is  $|mn\rangle = |00\rangle$  with the energy  $E_0 = \frac{3}{2}\hbar\omega_y$  and the next lowest eigenenergy state is  $|01\rangle$  with the energy  $E_1 = \frac{5}{3}E_0$ . The typical value of the energy spacing between the quantum dot levels,  $E_0$ , is of order  $1 - 10 meV$ . It can be several times larger in self-assembled dots. The energy scale of the Rashba term is  $E_R = c_R/R \sim 0.01 - 10 meV$ , where the length scale  $R \sim 100 \text{\AA}$  is the lateral dimension of the quantum dot. The energy scale of the distortion potential is  $E_p = \langle 0|\epsilon'y|1\rangle$ . Its magnitude is of order  $1 - 10 meV$ , depending on the electric field applied along the y-axis. The energy scale of the Dresselhaus term is  $E_D = c_D/R^3$ , and it can be larger or smaller than the Rashba term, depending on the material[19]. Here  $E_D = ic_D \langle 0|k_y|1\rangle (\langle f(z)|k_z^2|f(z)\rangle - \langle 0|k_x^2|0\rangle) = c_D/R_y R_z^2 - c_D/R_y R_x^2$ , and it originates from the second term of the Dresselhaus term (the first term in the Dresselhaus term is zero in our model since  $\langle 0|k_x|0\rangle = 0$ ).

We will use a truncated version of the Hamiltonian matrix, which makes it possible to write the eigenstates as a linear combination of four basis states made out of  $|mn\rangle$  and spin degree of freedom:

$$|\psi\rangle = c_{0,0,\uparrow}|0,0,\uparrow\rangle + c_{0,1,\uparrow}|0,1,\uparrow\rangle + c_{0,0,\downarrow}|0,0,\downarrow\rangle + c_{0,1,\downarrow}|0,1,\downarrow\rangle. \quad (12)$$

The advantage of this truncated model is that it is *exactly solvable*. In this paper we choose the following eigenstates of the lowest energy shell as the instantaneous basis vectors:

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{N_1} \begin{pmatrix} E_0 - \sqrt{E_0^2 + 9(E_D^2 + E_p^2 + E_R^2)} \\ 3(E_D - iE_R) \\ 0 \end{pmatrix}, \\ |\psi_2\rangle &= |\bar{\psi}_1\rangle. \end{aligned} \quad (13)$$

The non-Abelian vector potentials are evaluated with respect to these basis vectors.

In the following we choose the adiabatic parameters as the Rashba energy and the distortion energy

$$\begin{aligned} \lambda_1 &= E_R = c_R/(\sqrt{2}R_y) \\ \lambda_2 &= E_p = \langle 0|\epsilon'y|1\rangle = \frac{\epsilon'}{\sqrt{2}}R_y \end{aligned} \quad (14)$$

The cyclic adiabatic path is given by  $E_R(t) = E_{R,c} + \Delta E_R \cos(\omega t)$  and  $E_p(t) = E_{p,c} + \Delta E_p \sin(\omega t)$ .

## B. Geometric dependence

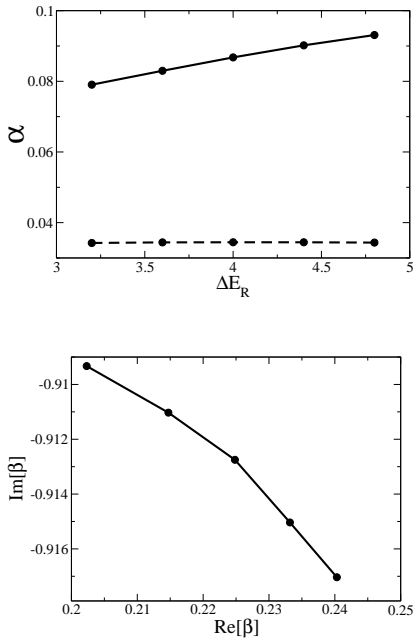


FIG. 5: For elliptic paths with fixed area with  $\Delta E_R$  changing from  $3.2E_0$  to  $4.8E_0$ . In the presence of both of the Dresselhaus and Rashba terms (solid line). Only the Rashba term present (dashed). How  $\beta$  changes with semiaxis  $\Delta E_R$  in the presence of both of the Dresselhaus and Rashba terms.

When only the Rashba term is present the structure of the non-Abelian vector potentials is remarkably simple. The orthonormalization  $\langle \psi_i | \psi_j \rangle = \delta_{ij}$  gives that the diagonal matrix elements  $(A_p)_{i,i}$  are real and that the off-diagonal elements satisfy  $(A_p)_{i,j} = (A_p)_{j,i}^*$ . We find non-Abelian vector potentials  $A_1$  and  $A_2$

$$A_1 = -\frac{E_p \left(1 + \frac{1}{\sqrt{9E_p^2 + 9E_R^2 + 1}}\right)}{2(E_p^2 + E_R^2)} \sigma_x, \quad (15)$$

$$A_2 = \frac{E_R \left(1 + \frac{1}{\sqrt{9E_p^2 + 9E_R^2 + 1}}\right)}{2(E_p^2 + E_R^2)} \sigma_x. \quad (16)$$

(Note that  $\langle f(z) | \frac{\partial}{\partial E_R} | f(z) \rangle = 0$ .) The diagonal elements  $A_{ii}$  are zero and the off-diagonal elements  $A_{ij}$  are real. From this it follows that the parameter  $\beta = 1$  and only one geometric number  $\alpha$  is needed. The matrix Berry phase is given by

$$\Phi_C(n) = \begin{pmatrix} \cos(\alpha n) & i \sin(\alpha n) \\ i \sin(\alpha n) & \cos(\alpha n) \end{pmatrix}. \quad (17)$$

One can show that  $[A_1, A_2] = 0$  when only the Rashba term is present, and that Stokes' theorem can be applied, Eq. (7). The field strength has a simple form  $F_{12} = f \sigma_x / E_0^2$  where

$$f = -\frac{9}{2(9x^2 + 9y^2 + 1)^{3/2}}, \quad (18)$$

with  $x = E_p/E_0$  and  $y = E_R/E_0$ . From this we can compute the rotation angle

$$\alpha = \int_A f dx dy, \quad (19)$$

which gives  $|\alpha| < 0.7854$ .

When both the Rashba and Dresselhaus terms are present Stokes' theorem is not applicable. We have investigated how  $\tilde{m}$  and  $\alpha$  depend on the shape of the paths. They can be computed numerically by solving the time dependent Schrödinger equation

$$i\hbar \dot{c}_i = - \sum_j A_{ij} c_j. \quad (20)$$

The matrix elements  $A_{ij}$  are given by  $A_{ij} = \hbar \sum_p (A_p)_{i,j} \frac{d\lambda_p}{dt}$ , where the sum over  $p$  in  $A_{ij}$  is meant to be the sum over  $\lambda_p$ . We have considered elliptic paths with the area  $\pi \Delta E_R \Delta E_p = 16\pi E_0^2$  with  $E_{R,c} = 5E_0$  and  $E_{p,c} = 5E_0$ . The results are shown in Fig.5. (Note that the dependence of  $E_D$  on the adiabatic parameter  $E_R$  is not well-known, but the essential physics does not depend on the exact functional form, as discussed in Ref.[11]). Fig.5 also shows numerical results for  $\alpha$  when only the Rashba term is present. We have *verified* that these numerical results agree well with the results obtained from the equation Eq.(19).

### C. Time dependence

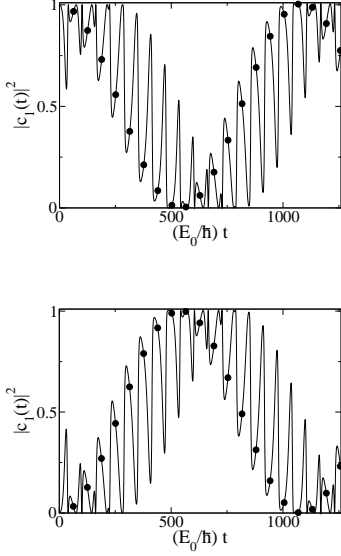


FIG. 6: Probabilities as a function of time in the presence of the Rashba spin orbit coupling. Black dots are fitted values at  $t = nT$  using the analytical expression for the matrix Berry phase, Eq.(17).

Let us first investigate the time evolution of the system in the presence of the Rashba term. The time-dependent Schrödinger equation is solved using the Runge-Kutta method. For the parameters  $E_{R,c} = 2E_0$ ,  $E_{p,c} = E_0$ ,  $\Delta E_R = 1.9E_0$ ,  $\Delta E_p = 0.9E_0$ , and  $\hbar\omega_1 = 0.1E_0$  we calculate  $|c_1(t)|^2$  and  $|c_2(t)|^2$ , see Fig.6. We can fit the data to  $|c_1(nT)|^2 = \cos^2(\alpha n)$ ,  $|c_2(nT)|^2 = \sin^2(\alpha n)$ , with  $\alpha \approx 0.182$ . In this case the matrix Berry phase takes a simple form with  $\beta = 1$ . We have *tested* our numerical method as follows: For two different periods  $T_1$  and  $T_2$  we find the matrix Berry phases are the same, i.e.,  $c_2(T_1) = c_2(T_2)$  and  $c_1(T_1) = c_1(T_2)$ . This is consistent with the fact that the matrix Berry phase is a geometric effect independent of how the path is parameterized. Let us now include the Dresselhaus spin orbit term as described in Ref [11]. In this case the matrix Berry phase is more complicated since  $\beta \neq 1$ . The diagonal elements  $A_{ii}$  are non-zero and the off-diagonal elements  $A_{ij}$  are complex numbers. Using the same parameters  $E_{R,c}$ ,  $E_{p,c}$ ,  $\Delta E_R$ ,  $\Delta E_p$ , and  $\omega$  as in the previous calculation, we have computed  $|c_1(t)|^2$  and  $|c_2(t)|^2$ , which are displayed in Fig.7. These results can be fitted with the parameters  $\alpha \approx -0.477$  and  $\beta \approx -0.190 + i0.965$ . In both cases the numerical results are consistent with those of the analytical expression at  $t = nT$ , given by Eq.(8). In the time

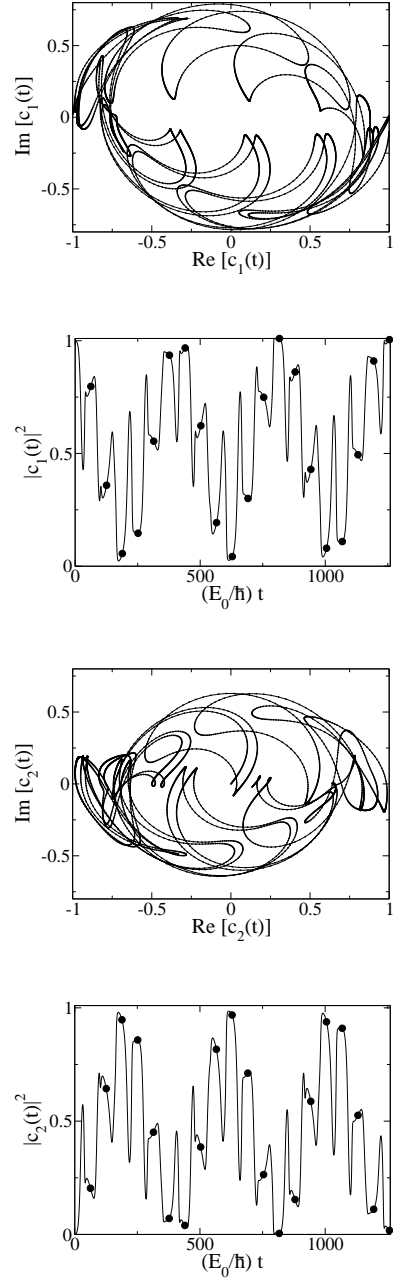


FIG. 7: Probability  $c_1(t)$  when Dresselhaus spin orbit interaction is included. Black dots are fitted values using the analytical expression for the matrix Berry phase, Eq.(8). The initial state is  $c_1(0) = 1$  and  $c_2(0) = 0$ .

interval  $(n-1)T < t < nT$  the coefficients  $c_{1,2}(t)$  display a complicated behavior. It should be stressed that even when the adiabatic path is *not* closed the actual state  $\Psi(t)$  is given by a nontrivial linear combination of the instantaneous eigenstates  $\psi_1(t)$  and  $\psi_2(t)$ . In this case a matrix phase of the form

$$\Phi_C = P e^{i \int_C \sum_p A_p d\lambda_p}, \quad (21)$$

will appear, where the path  $C$  is unclosed.

### III. DISCUSSIONS AND CONCLUSIONS

We have investigated II-VI and III-V n-type semiconductor dots with spin orbit coupling terms and have shown in this paper that the geometric information of a closed adiabatic path can be encoded in the rotation axis and angle of the spin half matrix. This result is applicable even for strongly correlated states, as along as the states are doubly degenerate. In our truncated model for the Hamiltonian the matrix Berry phase can be found *exactly* when the Rashba term is present. In this case the Stokes' theorem is applicable and the relevant field strength can be calculated analytically.

Even when the adiabatic path is *not* closed the actual state,  $\Psi(t)$ , at time  $t$  will not be the instantaneous eigenstate  $\psi_i(t)$ . Instead it is a linear combination of the instantaneous eigenstates

$$|\Psi(t)\rangle = c_1(t)|\psi_1(t)\rangle + c_2(t)|\psi_2(t)\rangle. \quad (22)$$

The expansion coefficients  $c_i(t)$  change in a non-trivial way in the interval  $(n-1)T < t < nT$ , see Figs.6 and 7.

Experimental investigations of matrix Berry phases in II-VI and III-V semiconductor quantum dots would be most interesting. Optical dipole measurements[22] were proposed before[11]. Here we propose a transport measurement. In order to measure the matrix Berry phase we perform the following set of manipulations:

1. Prepare an initial state by applying a small magnetic field and taking the zero field limit, as explained in detail in Ref. [11].
2. Perform an adiabatic cycle by changing  $E_R$  and  $E_p$  that defines the shape of the electric confinement

potential, see Fig.4(a).

3. Change suddenly the electric field along the z-axis so that the electron can tunnel out of the dot, see Fig.4(b).
4. Measure the spin dependence of the tunneling current[2].
5. Repeat this set of procedures many times.

After this procedure we repeat the entire scheme one more time except the item 2. The difference between spin-up (down) currents of these two schemes is a demonstration of the presence of the matrix Berry phase.

Our truncated model can be applied to self-assembled quantum dots[23], which have large energy shell differences. In these dots exchange and correlation effects can be neglected[12], and it would be sufficient to consider only the electron in the last occupied energy shell. On the other hand, in gated quantum dots[24] with smaller energy differences between the shells and with odd number of electrons correlation effects may be relevant. A computational scheme to calculate the matrix Berry phase in such a system is developed in Ref.[12].

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